Seismic inelastic torsional analysis by the modified substitute structure method

Ken S.K.Tam
Loewen Bogdonov Associates, Vancouver, British Columbia, Canada
D.L.Anderson, N.D.Nathan & S.Cherry
Department of Civil Engineering, University of British Columbia, Vancouver, Canada

ABSTRACT: An efficient three dimensional method of analyzing structures for seismic inelastic reponse is described. The method uses three degrees of freedom per storey and an iterative elastic modal analysis, where the stiffness and damping of the elastic model is varied in accordance with the ductility demand.

1 INTRODUCTION

In the seismic analysis of many types of buildings, the dictates of prudence, as well as the code, require that one make a full three dimensional or torsional analysis of the structure. Yet, even for an elastic analysis, the models can become very large and expensive to run, while an elastic-plastic analysis is out of the question for many structures. The analysis method described in this paper (Tam 1985) provides an efficient procedure for determining the elastic response, as well as an approximate non-linear elastic-plastic response of a structure useful for predicting the ductility demand and the displacements. It is a combination of an elastic torsional response analysis in which the floors are assumed to be rigid diaphragms, so that the dynamic problem is reduced to three degrees of freedom per floor (MacKenzie 1974), and the modified substitute structure method (Yoshida 1979, Metten 1981, and Hui 1984).

The modified substitute structure method has been shown to provide good estimates of the ductility demand in plane buildings. The method is an iterative one using elastic modal analysis and the response spectrum approach to predict the non-linear or plastic reponse of the structure. Since it uses modal methods with a response spectrum it is much more economical than a full elastic-plastic time-step analysis. It also has the advantage of working with a response spectrum envelope rather than with several individual or synthetic earthquake records.

The method has been applied to eccentric five storey frame buildings and five storey coupled wall buildings. The results of the analysis have been compared to those obtained with the program DRAIN-TABS, which makes a full three dimensional elastic-plastic time-step dynamic analysis. The comparison shows very good agreement in both displacements and ductility demand between the proposed method and the average DRAIN-TABS results for four different earthquake records.

2 MODIFIED SUBSTITUTE STRUCTURE METHOD

The modified substitute structure method is an extension of the substitute structure method originally developed for the design of concrete structures (Shibata and Sozen 1976). Their original thesis was that for a certain desired ductility factor in a building component, the design force level could be determined by carrying out an elastic dynamic analysis on a structure in which the stiffness of that component had been reduced by the desired ductility factor, and where the damping had been adjusted to reflect the corresponding level of inelastic response. The modified substitute structure method uses the same approach, except that it is applied to a structure with known properties, to determine the ductility demand under a given level of seismic excitation. To do this an iterative approach is used in which the stiffness and the damping of each member are continually changed to reflect the level of ductility demand, until the internal force agrees with the strength of the member.

The original work on the modified substitute structure method was for plane frames (Yoshida 1979). He showed that some of the restrictions required by Shibata and Sozen could be relaxed, and that for frames with more than one bay, and especially for frames with strong columns and weak beams, the ductility demand predicted by the method was in very good agreement with predictions made using DRAIN-2D, a non-linear time-step plane frame program. The method was then extended to coupled shear walls (Metten 1981) and it was shown that it again gave good predictions of the ductility requirements and deflections.

In the description of the method it is convenient to use a factor called the damage ratio rather than the ductility factor, although for members which are elastic perfectly plastic, i.e. with no strain hardening, the two factors are identical. In the original definition of the method the damage ratio for each member was defined as the effective stiffness of the member divided by its original elastic stiffness, i.e. the flexural stiffness in any one iteration is

$$EI_{si} = EI_{ai}/\mu_i$$

where

EI_{si} = cross-sectional flexural stiffness of the i-th member in the substitute structure

EI_{ai} = cross-sectional flexural stiffness in the actual frame

μ_i = damage ratio of the i-th member

Using the reduced member stiffness, natural periods and mode shapes are obtained from a linear dynamic analysis. To determine the member forces from a response spectrum it is necessary to know the damping for each mode. This is done by using the member forces and resulting strain energy from the previous iteration as a weighting factor to determine the amount of damping that each member contributes to the modal damping value. The damping in each member is related to the damage ratio in that member, and is given by

$$\beta_i = 0.02 + 0.2(1 - 1/\sqrt{\mu_i})$$

This relation, for reinforced concrete members, was determined by back analyzing the response of simple structures subjected to shaking table tests (Gulkan and Sozen 1971).

The damping ratio for each mode, called the smeared damping ratio, is then given by

$$\beta_m = \frac{\sum_i (P_i{}^m \beta_i)}{\sum_i P_i{}^m}$$

where

$P_i{}^m$ = relative flexural strain energy in the i-th member for the m-th mode

$$= \frac{L_i}{6\,EI_{si}}\left[(M^m_{ai})^2 + M^m_{ai}M^m_{bi} + (M^m_{bi})^2\right]$$

L_i = member length

M^m_{ai}, M^m_{bi} = bending moments at the ends of the substitute structure member for the m-th mode

The expression for $P_i{}^m$ is a measure of strain energy and each member is assumed to contribute to the modal damping in proportion to the relative value of strain energy it absorbs in that mode.

Once the modal damping values are calculated the modal excitation can be found from a response spectrum and the member forces in each mode can be determined. The member forces from the modes are combined and compared to the member strengths. If they are not in agreement the damage ratios are altered and another iteration carried out. Strictly speaking the dynamic equations of motion cannot be separated into uncoupled modal equations unless the damping matrix is a linear combination of the mass and stiffness matrices, and the modal damping values as calculated above may not satisfy this requirement. However, since the damping values are small this is assumed not to be of importance.

As described above the reduction of the member flexural stiffness by the damage ratio implies that both the damage ratio and stiffness are the same at the two ends of the member, which will generally only be true for beams in a frame of strong columns and weak beams. The method has been extended by considering the plastic deformation to consist of a plastic hinge at each end of the member (Hui 1984). As a result damage ratios are calculated for each end and then used in a more complex manner to calculate the member stiffness and contribution to damping. In the results that follow Hui's procedure is used.

In a torsional analysis it often occurs that there are several modes with very closely spaced periods. Summing the modal effects using the square root sum squares (SRSS) or the absolute sum method proved inadeqate in these cases and the complete quadratic combination (CQC) method was found to give superior results.

3 STRUCTURAL IDEALIZATION

Because torsion analysis requires analysis of the entire structure in three dimensions it is imperative that some simplifying assumptions be made to reduce the size of the analysis problem. For many structures, especially tall office and apartment type structures, the floors are essentially rigid in their plane when compared to the lateral displacements. For such structures the analysis can be reduced to three degrees of freedom per floor, two horizontal displacements and a rotation. The assumptions necessary for such a reduction are: the floors are rigid in their own plane, the masses are concentrated at the floor levels, and there are negligible vertical inertial forces.

MacKenzie used the above approach to calculate the elastic torsional response of buildings. He considered the structure to be made up of a series of frames or walls connected to rigid diaphragms. With this assumption each frame has two degrees of freedom per joint, a vertical displacement and a rotation, plus one lateral translation degree of freedom per floor. If the frames are independent of each other, i.e. they do not intersect at common elements, then the vertical and rotational degrees of freedom can be removed by static condensation resulting in a single degree of freedom at each floor level.

When frames have a common column it is important that compatibility with regard to vertical displacements at the common column be enforced. This is done by keeping the vertical degrees of freedom in the common columns until the frame stiffnesses are assembled into the structure stiffness matrix. At this stage compatibilty can be enforced and the vertical degrees of freedom removed by static condensation, to give the structure stiffness matrix with three degrees of freedom, two translations and a rotation, per floor. The mass matrix is easily calculated and the mass and stiffness matrices then used to determine the mode shapes and natural frequencies.

When two frames meet at a right angle the rotations in the two frames at the common column are at right angles and are independent, provided the torsional resistance of individual members is neglected. However, if the frames do not meet at right angles the two rotations are not independent, and additional compatibility constraints are required. This feature was not included in the program but all the examples considered had the frames meeting at right angles, and so the omission was irrelevant.

4 EARTHQUAKE INPUT

The accuracy of the modified substitute structure method was determined by comparison with an elastic-plastic time-step analysis, which requires as input an earthquake time history. The Shibata and Sozen earthquake spectrum A represents the smoothed average spectrum of six records, four of which are the two components of the 1940 El Centro record and the two components of the 1952 Taft record. Figs. 1 and 2 show spectrum A and the spectra of each of the four records for two different values of damping. It was considered sufficient to use only the four mentioned records when comparing the time-step analysis with the modified substitute structure method results from spectrum A.

The spectral values for 2% damping are given in Fig. 1. For values of damping other than 2% the following relation was used to calculate the spectral accleration:

$$S_a = S_{a2}\,[8/(6+100\beta)]$$

where

S_{a2} = spectral acceleration for 2% damping ratio
β = damping ratio

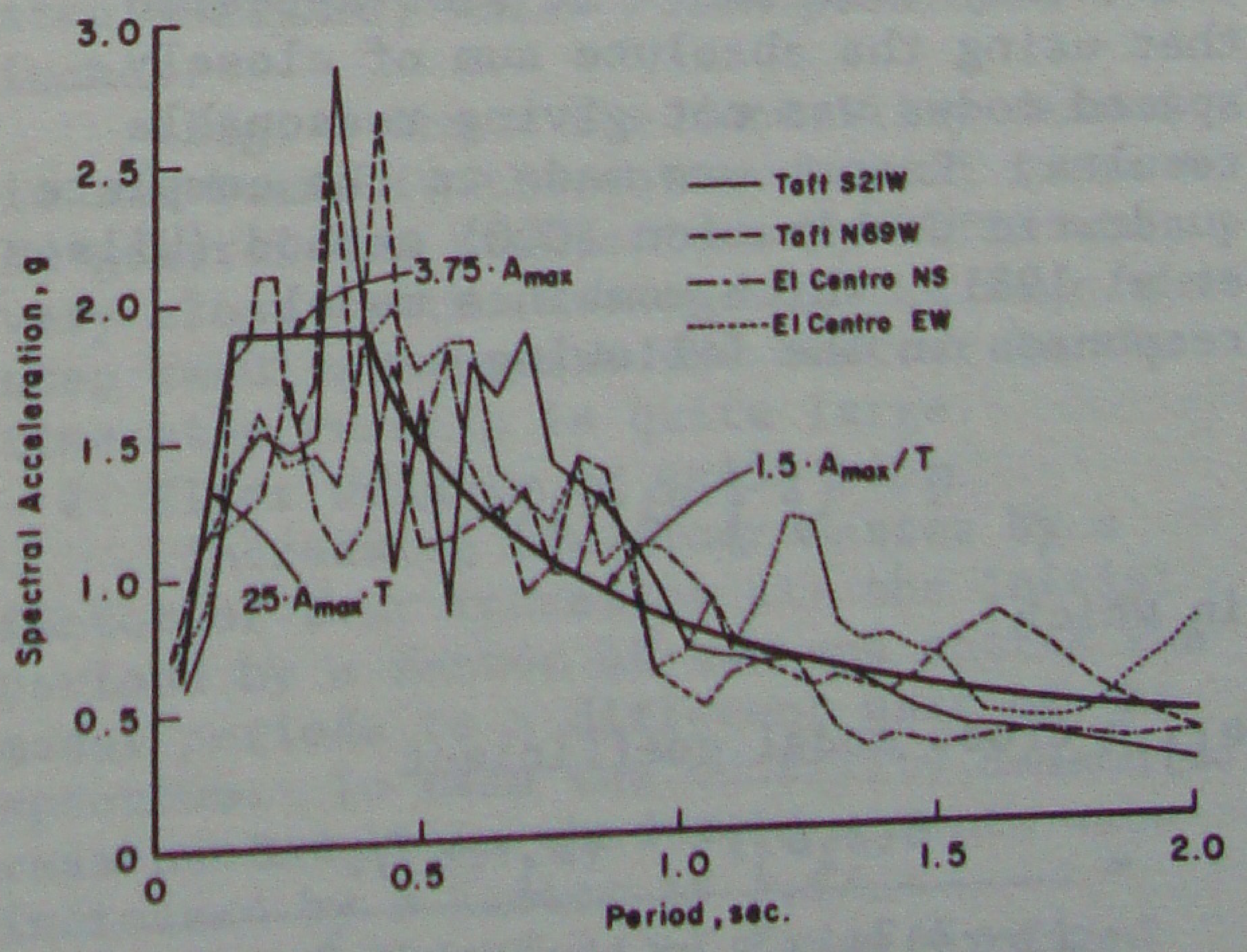

Figure 1. Response spectra. 2% damping. 0.5 g peak ground acceleration.

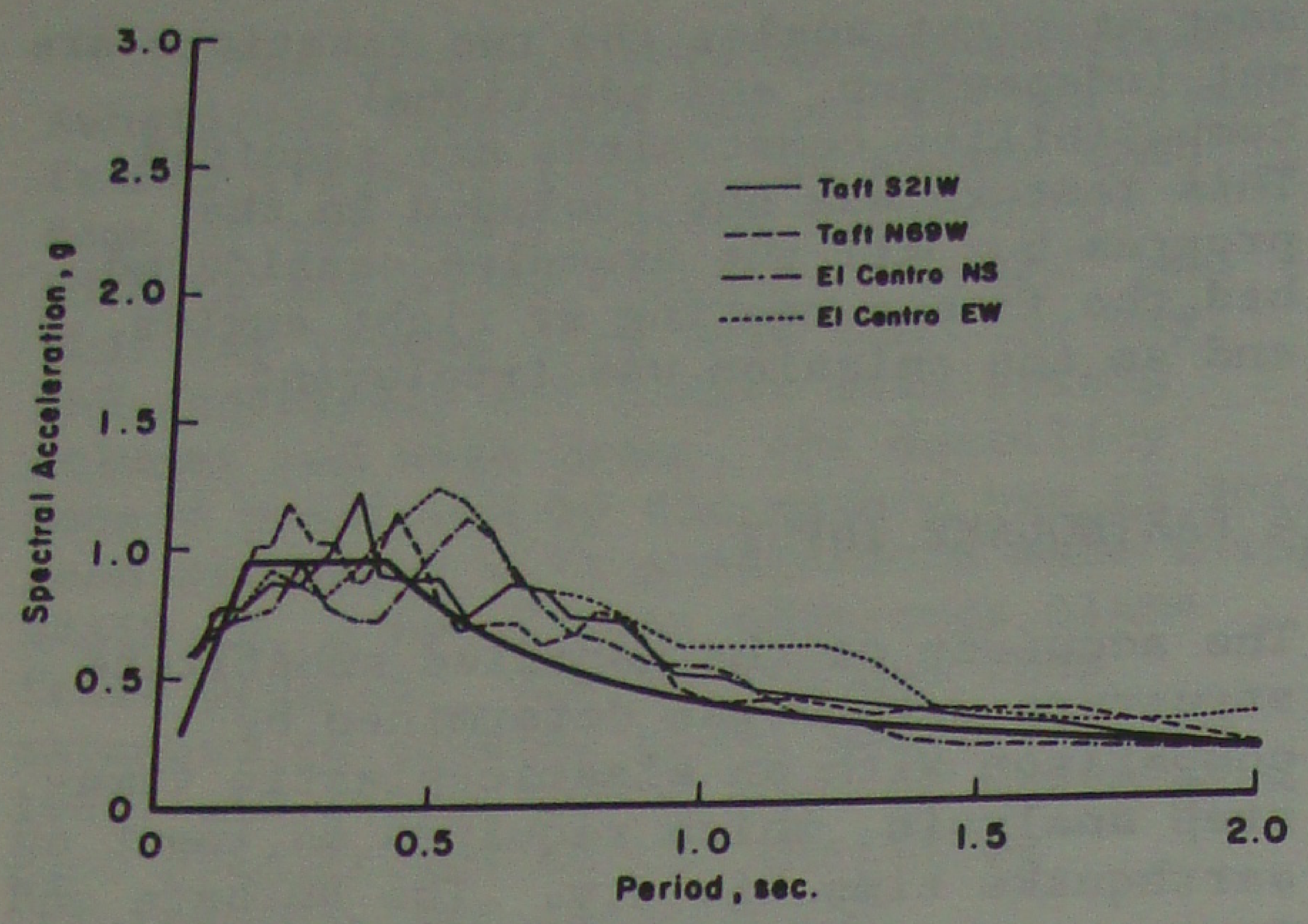

Figure 2. Response spectra. 10% damping. 0.5 g peak ground acceleration.

5 Results

The program resulting from the combination of the modified substitute structure method and the three dimensional structural idealization has been named PITSA (Pseudo Inelastic Torsional Structural Analysis). The elastic torsional response provided by PITSA was tested on both a frame type and a shear wall type building by comparing the results with ETABS, a well documented three dimensional modal analysis program developed at the University of California, Berkeley. The frequency and force results were nearly identical despite some small differences in the programs.

5.1 Complete quadratic combination method

Early in the work it became obvious that the SRSS method for combining modal responses was inadequate because of the very closely spaced modal frequencies that inevitably occurred. It also appeared that using the absolute sum of closely spaced modes was not giving reasonable results. Resort was made to the complete quadratic combination (CQC) method (Wilson et al 1981), which combines modal responses in the following way:

$$Q = \left(\sum_i \sum_j \rho_{ij} Q_j Q_i\right)^{1/2}$$

in which

ρ_{ij} = cross-modal coefficient

$$= \frac{8(\beta_i \beta_j)^{1/2}\,(\beta_j + r\beta_j)\,r^{3/2}}{(1-r^2)^2 + 4\beta_i\beta_j r(1+r^2) + 4(\beta_i^2+\beta_j^2)r^2}$$

where

Q_i = maximum contribution of the i-th mode to the response of interest
β_i = i-th mode damping
r = ratio of modal periods, T_i/T_j.

The cross-modal coefficient approaches unity for modes with close periods but is very small for modes with small damping and widely separated periods.

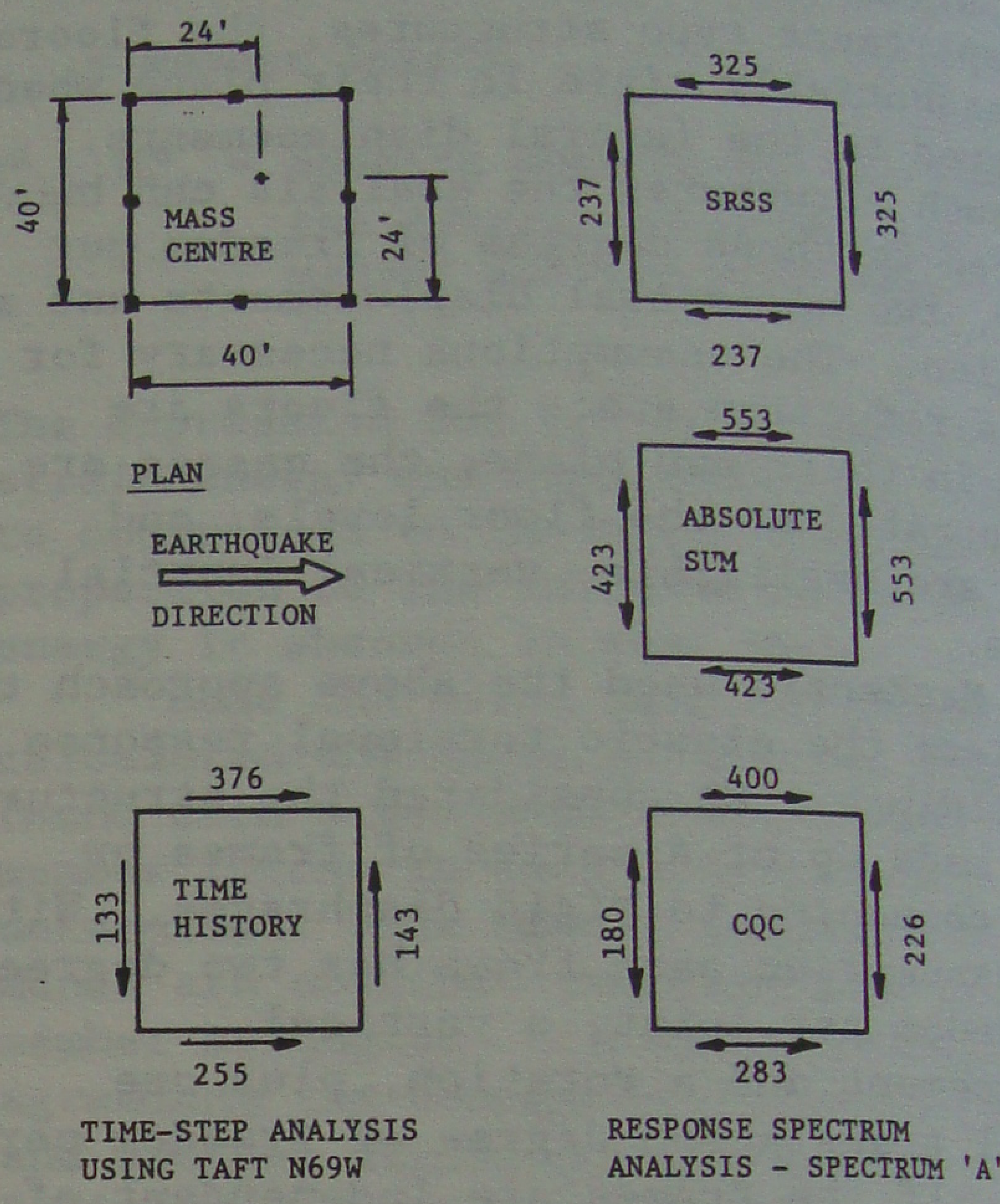

Figure 3. Comparison of modal combination methods.

To compare the different methods of combining modal responses a simple five storey elastic building with eccentric masses at each floor level (see Fig. 3) was analyzed using the modal method with spectrum A, and then compared with the results of a time step analysis using the Taft N69W record. Although the spectrum from this record does have quite sharp peaks and valleys, for the periods of interest it is in good agreement with spectrum A. The first two periods of the trial structure, as well as the fourth and fifth, are very close to one another, and so it provides a good test of cross-modal coupling. Fig. 3 gives the base shears in each frame for the different methods. Assuming the time step analysis to give the correct response, it is clear that although the complete quadratic combination method overestimates the base shears in the frames perpendicular to the direction of the base motion, it is much closer than the other two methods, especially the absolute sum method, which is generally used when there are closely spaced periods.

The complete quadratic combination (CQC) method was used in all further comparison work reported here.

5.2 Testing for inelastic response

The time-step program DRAIN-TABS developed at the University of California, Berkeley, was used for comparison purposes. This program also idealizes the structure as a series of plane frames interconnected by a series of rigid horizontal diaphragms, although it does not enforce vertical compatibility at common columns as does PITSA.

In both programs the cracked moment of inertia for columns with axial load was assumed to be half the gross moment of inertia, and for beams one third the gross moment of inertia. In addition to the hysteretic damping, a viscous damping ratio of 2% was assumed for DRAIN-TABS, and taken as proportional to the tangent stiffness.

Comparisons between PITSA and DRAIN-TABS will be made on the basis of maximum displacements and member ductility demand. Member ductility is defined here in terms of member end rotation, which is calculated by summing the plastic hinge rotation and the yield rotation and then dividing by the yield rotation. The yield rotation is defined as the end rotation, based on a cracked section modulus, when the member is subjected to equal anti-symmetric end yield moments. In comparing member ductilities only the larger value of the ductility at the ends of the member is reported. The above definition provided a simple and reliable method of comparing ductility.

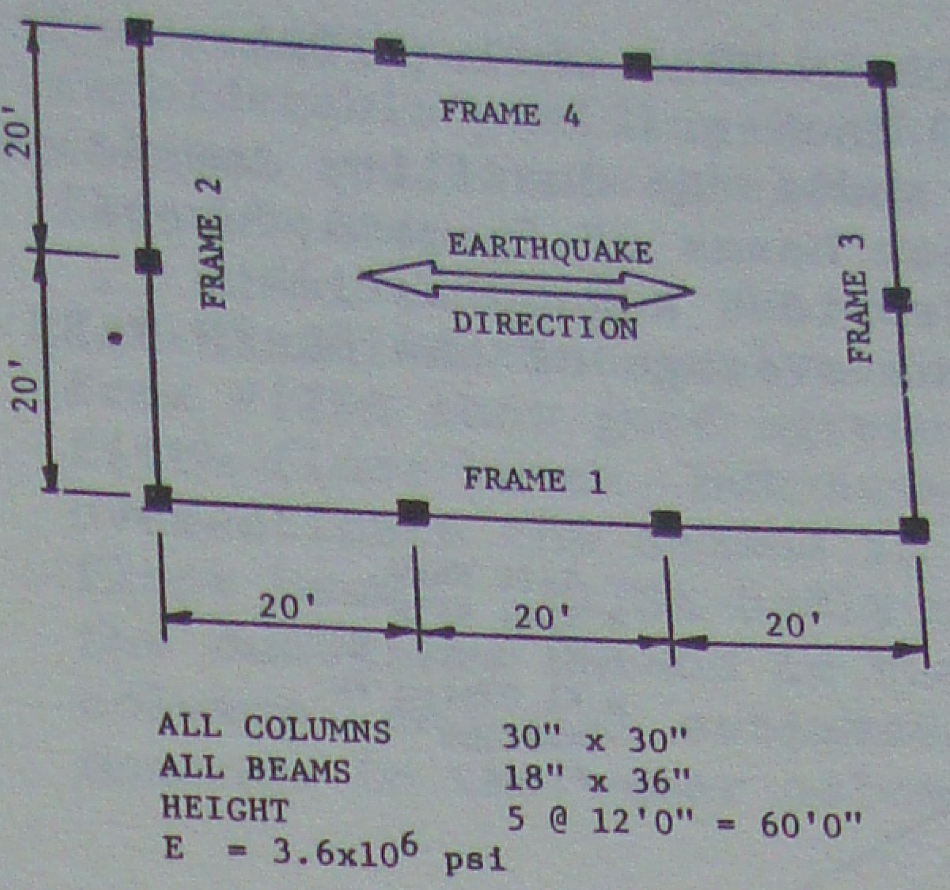

Figure 4. Dimensions and properties of the five storey frame building.

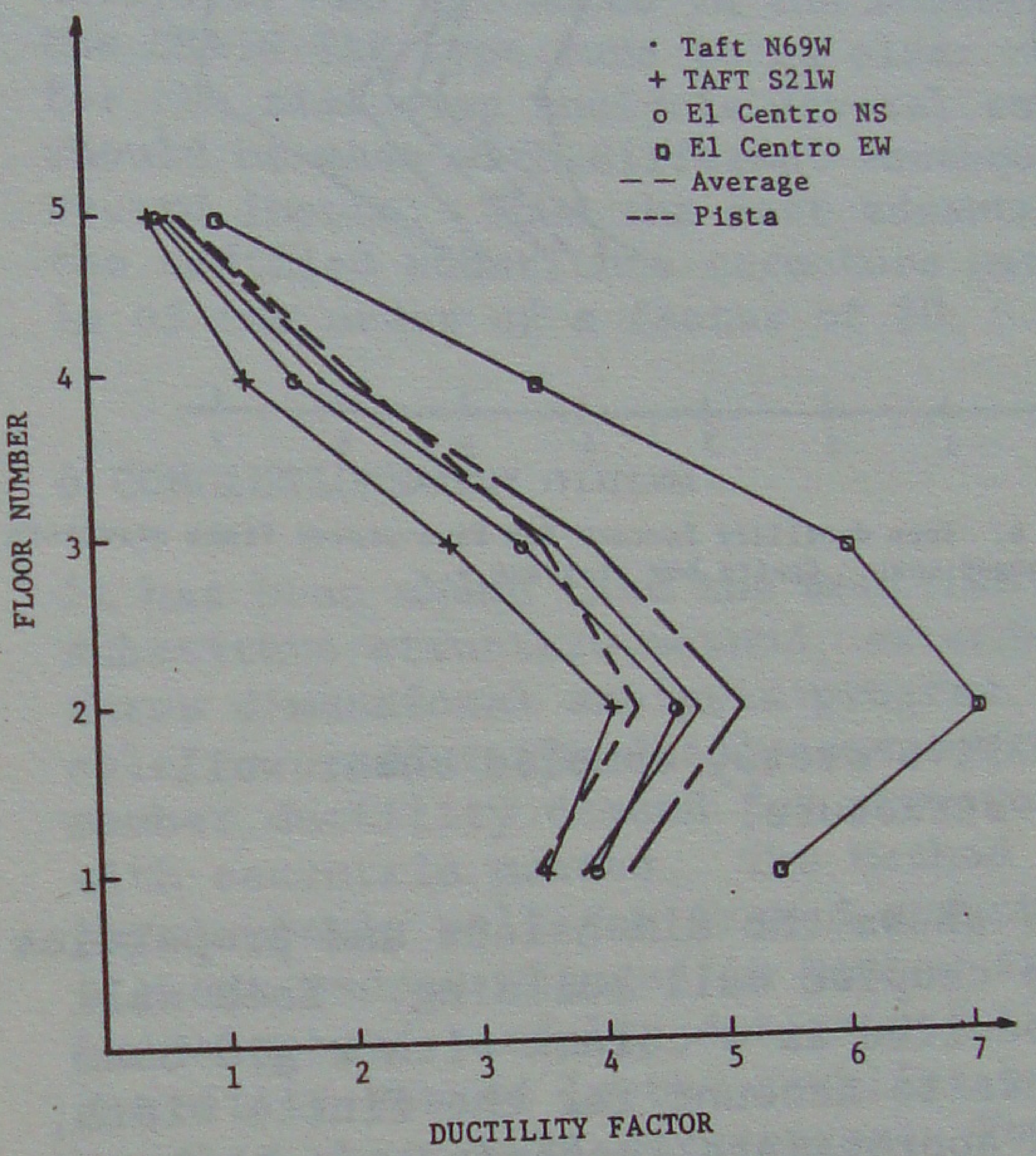

Figure 5. Beam ductility factors for five-storey frame structure. Centre bay of frame 4.

5.2.1 Five storey frame structure

Fig. 4 gives dimemsions and member sizes for the five storey frame test structure. The eccentricity of the mass occurs only in the top two floors and is 4 feet towards frame 4.

1. Floor weights of 130 kips.

For this combination of mass and stiffness the lowest period of the modes whose motion was mainly in the direction of the earthquake was 0.42 secs. for the structure in the initial cracked condition, while it was 0.66 secs. for the substitute structure with its final stiffness.

The deflection at the top of frame 4 ranged from 2.8 to 5.0 inches, with an average of 3.6 inches, for the four time step analyses, while PITSA predicted 3.8 inches.

Fig. 5 shows the ducutility demand in the beams in the center bay of frame 4. In general the predictions by PITSA are very close to the average of the four time step results, whereas the scatter in the time step results is quite large.

2. Floor weights of 4x130 kips.

Increasing the floor masses by a factor of four increases all the initial periods by a factor of two and shifts the modal periods to a different part of the spectrum. To keep the ductility demand to reasonable values the yield moments were increased by a factor of 1.67.

Top deflections of frame 4 ranged from 5.0 to 9.0 inches, averaging 6.7

inches, for DRAIN-TABS while PITSA predicted 8.0 inches.

Fig. 6 shows the ductility demand in the central bay beams of frame 4, where again PITSA provides a good estimate compared to the average of the DRAIN-TABS results.

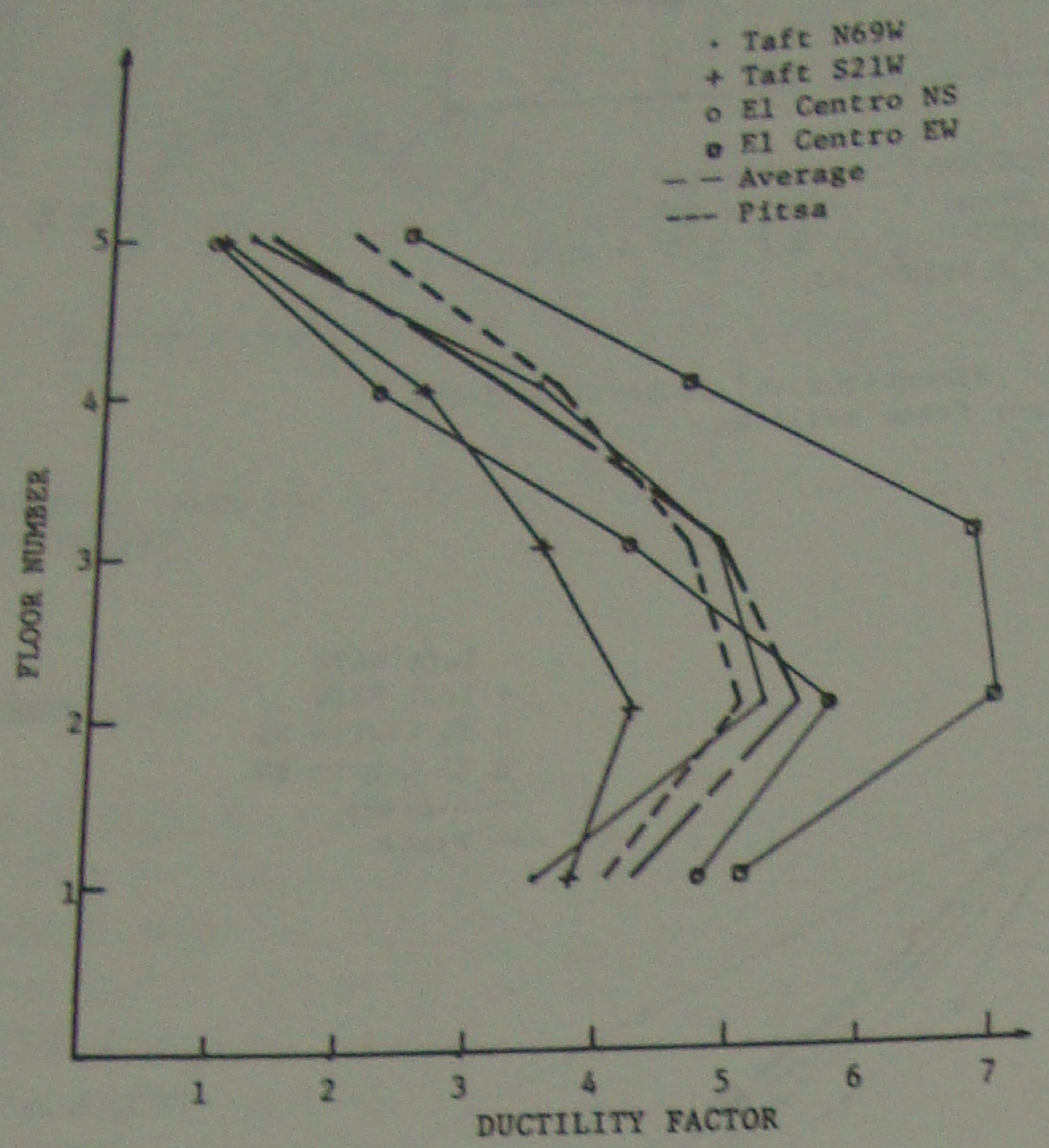

Figure 6. Beam ductility factors for five-storey frame structure with larger mass. Centre bay of frame 4.

5.2.2 Five storey coupled shear wall structure.

Fig. 7 shows the dimensions and properties of the coupled wall building. Each wall was modelled as a column with rigid beam elements to account for the finite width, and if appropriate connected to another wall with coupling beams to form a plane frame. The eccentricity of the mass in the top two floors was two feet towards wall 4.

1. Floor weights of 300 kips.

The fundamental period with the initial cracked section moduli was 0.38 secs.

The deflection of the top of wall 4 ranged from 0.9 to 1.4 inches with an average of 1.1 inches in the time step analysis, whereas PITSA predicted 1.5 inches.

Fig. 8 plots the ductility demand in the coupling beams in wall 4. Again PITSA provides a good estimate when compared to the average of the DRAIN-TABS results.

2. Floor weights of 4x300 kips.

Time step deflections ranged from 2.0 to 3.0 inches with an average of 2.4 inches compared to the PITSA prediction of 3.4 inches.

Despite the rather poor prediction of displacement Fig. 9 shows good agreement between PITSA and the DRAIN-TABS average for the the coupling beam ductility demand.

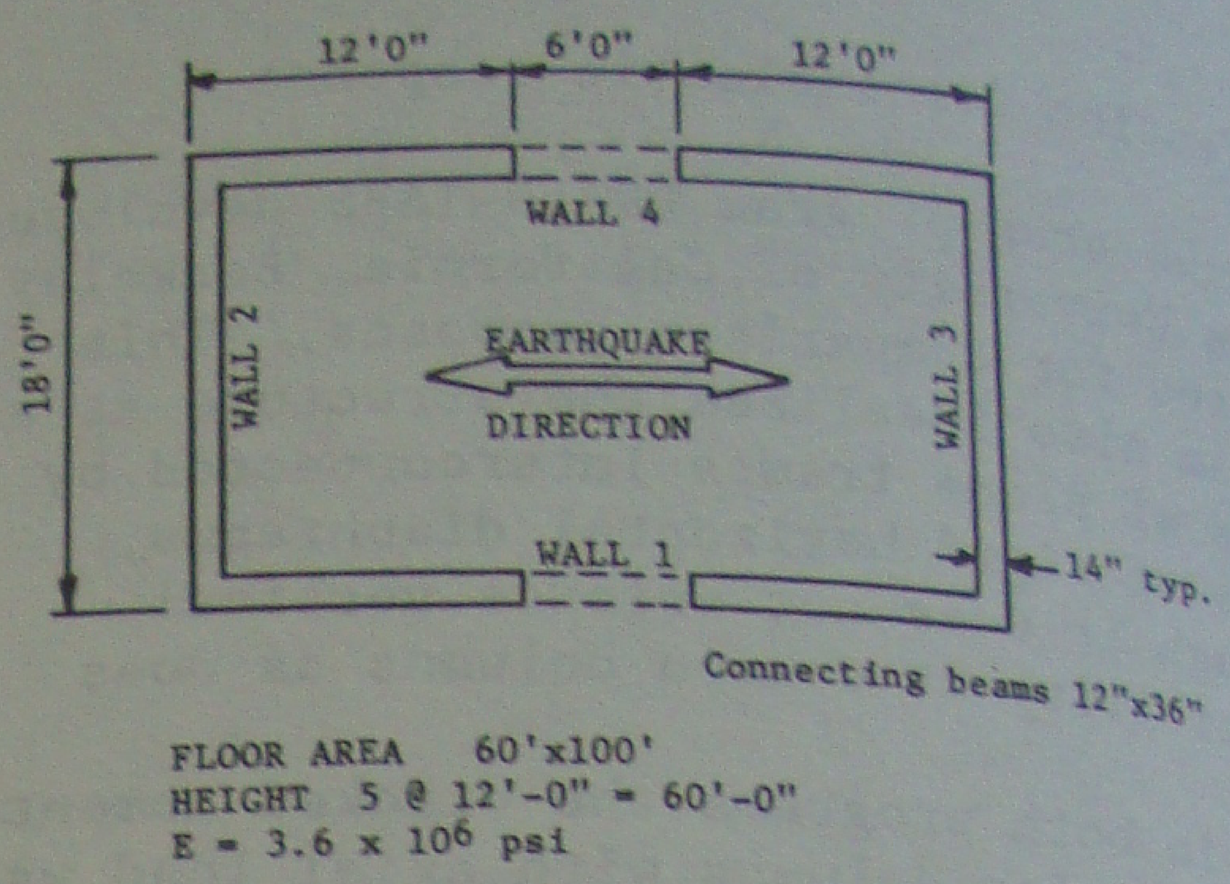

Figure 7. Dimensions and properties of the five storey coupled wall building.

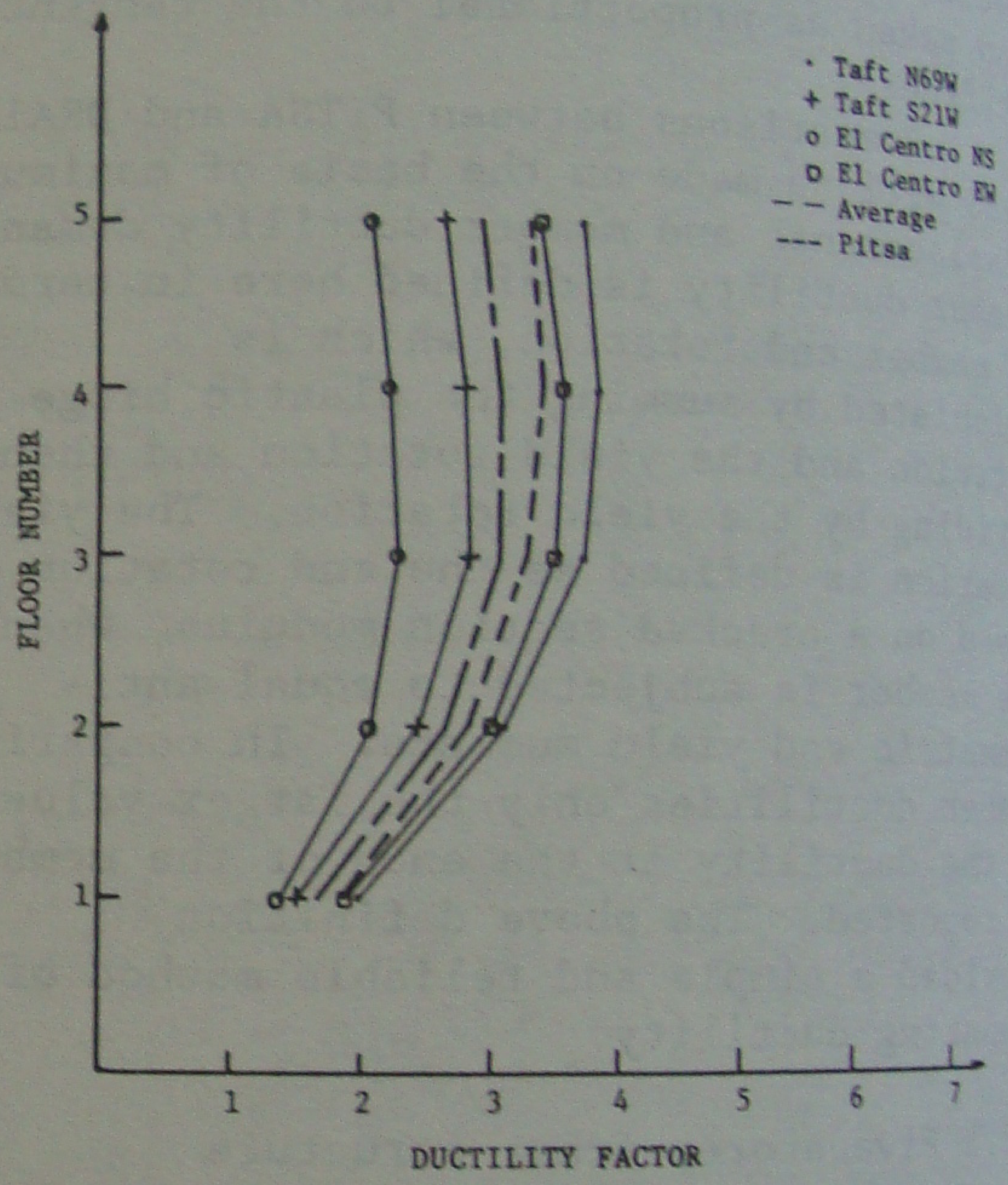

Figure 8. Connecting beam ductility factors for five-storey coupled wall structure.

In all of the examples discussed above the column and wall momemts predicted by PITSA were close to the DRAIN-TABS average and all were less than the yield moments. The best agreement occurred in the frame structures where the results were almost identical, to errors of up to 25% in the base moments of the second wall example. In the wall examples PITSA always provided a conservative estimate of the maximum moment, which is consistent with the prediction of larger displacements in the wall structures.

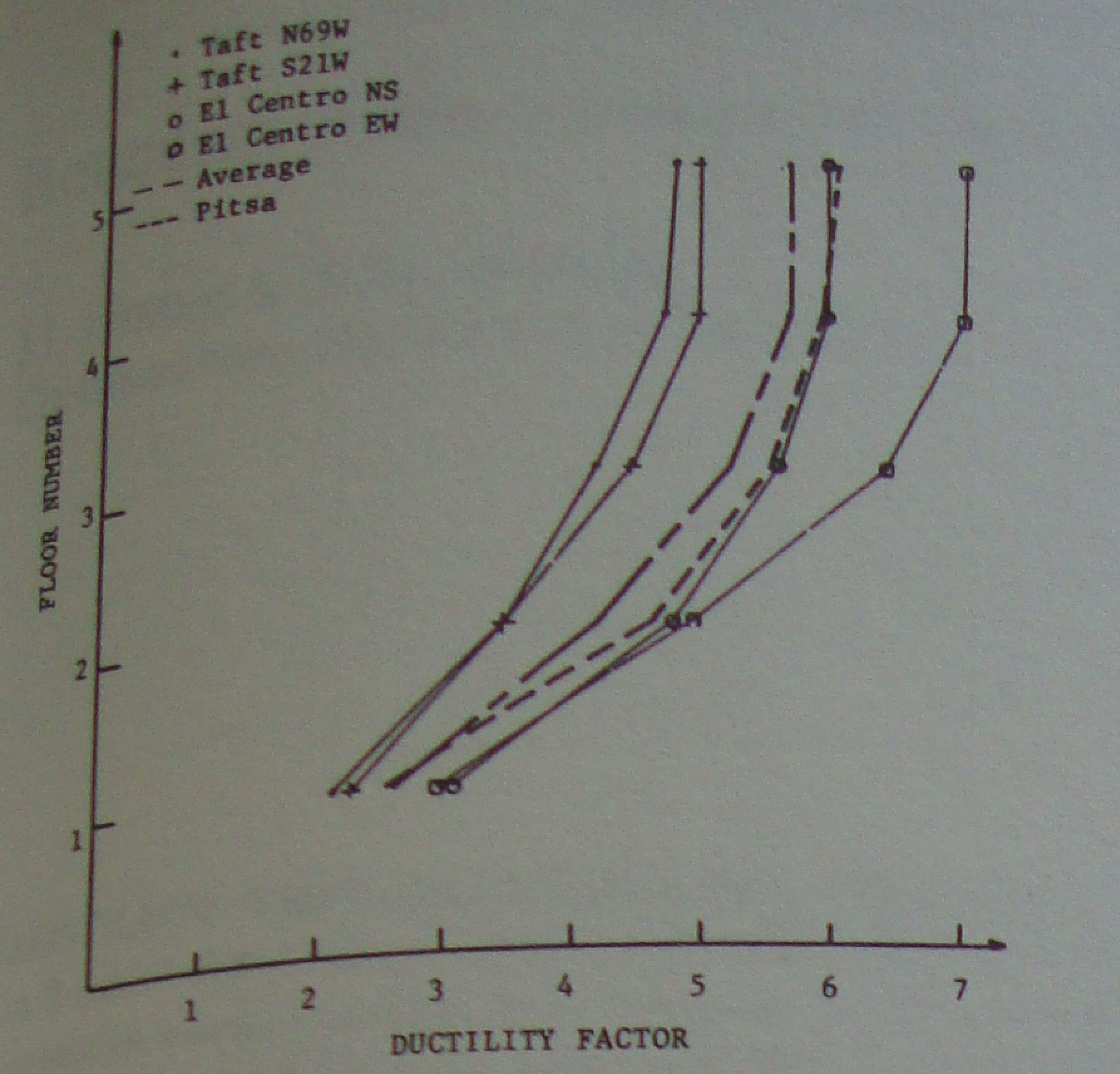

Figure 9. Connecting beam ductility factors for five-storey coupled wall structure with larger mass.

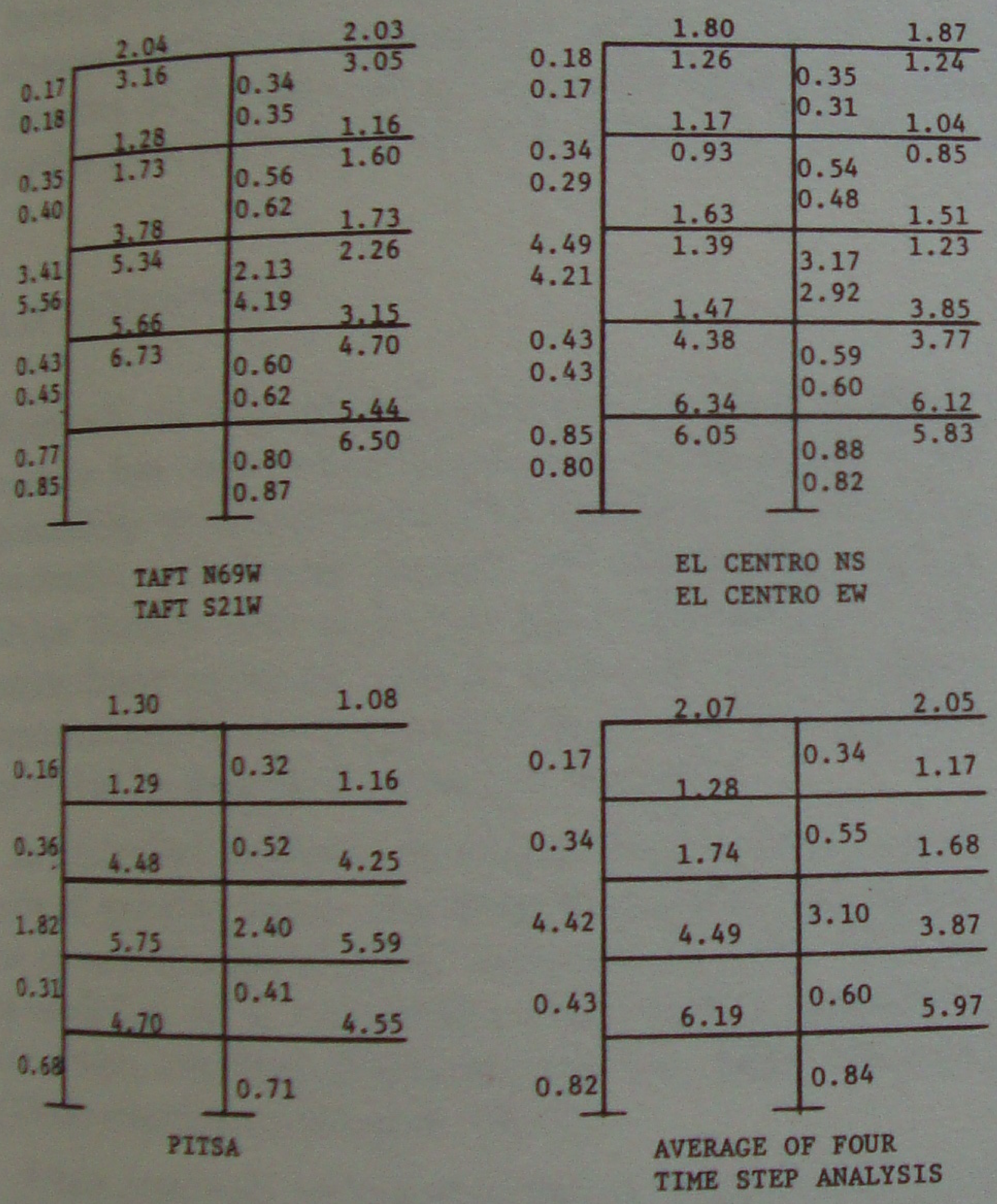

Figure 10. Ductility factor for five storey frame structure. Strong 4th floor beams, weak 3rd floor columns. Frame 4.

5.2.3 Five storey frame with some weak columns and strong beams.

Structures with large changes in stiffness or mass often require a dynamic analysis. To test the capability of PITSA to predict the ductility demand in such cases the frame of Fig. 4, with the increased mass, was analyzed with the yield moment capacity of the third floor columns reduced by 75% and the fifth floor beams increased by 100%. This resulted in considerable yielding in the third floor columns and little yielding in the fifth floor beams.

Fig. 10 gives the ductility demands for the different analyses. The results from PITSA show good agreement for the fifth floor beams, but severely overestimate the demand in the fourth floor beams. PITSA badly underestimates the ductility demand in the third floor columns, and also estimates smaller moments in the other columns.

5.3 Computer costs

The average CPU time taken to run one DRAIN-TABS analysis was 4.6 times a PITSA analysis. As evidenced in the scatter of the DRAIN-TAB type runs it is clear that for the time step analysis several runs should be made with different earthquake record inputs. Thus the cost advantage of the modified substitute structure method is of the order of a factor of 20.

6 CONCLUSIONS

It has been shown that the modified substitute structure method, extended to a three dimensional analysis program, is able to predict the displacements and member ductility demand for structures with eccentric masses. The method worked particularly well for framed structures without sudden changes in member strength, but also successfully determined the ductility demand in the coupling beams of coupled shear wall structures. For the one structure tested with one storey of weak columns and one storey of strong beams, the predicted ductility demands were in considerable error although the correct pattern of damage through the structure was predicted.

For the five storey structures analyzed the modified substitute structure method used about one-quarter to one-fifth the computer CPU time than did one analysis using a time step program. Thus the savings in CPU time could be in the order of 20 times if several runs have to be made for each time step analysis.

Since the modified substitute structure method statically condenses the stiffness matrix of each frame to a few degrees of freedom, and then assembles all the frame stiffnesses into a structure stiffness matrix having only three degrees of freedom per storey, it is possible to do a three dimensional analysis of quite large structures on the micro-computers found in many design offices. The program as developed has a generator to produce frame joint and member data if the frames are reasonably straightforward.

As proposed the method is appropriate for concrete structures but it is expected that it could be extended to steel frames by changing the expression for damping.

As a research tool it is now planned to use the method to investigate the effect of plasticity on the response of eccentric structures with different layouts of strength and stiffness.

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